Scaling behavior for finite $\mathcal{O}(n)$ systems with long-range interaction

Hassan Chamati* and Nicholay S. Tonchev†
Institute of Solid State Physics, 72 Tzarigradsko Chaussée, 1784 Sofia, Bulgaria
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A detailed investigation of the scaling properties of the fully finite $\mathcal{O}(n)$ systems, under periodic boundary conditions, with long-range interaction, decaying algebraically with the interparticle distance r like $r^{-d-\sigma}$, below their upper critical dimension, is presented. The computation of the scaling functions is done to one loop order in the nonzero modes. The results are obtained in an expansion of powers of $\sqrt{\varepsilon}$, where $\varepsilon=2\sigma-d$ up to $\mathcal{O}(\varepsilon^{3/2})$. The thermodynamic functions are found to depend upon the scaling variable $z=RU^{-1/2}L^{2-\eta-\varepsilon/2}$, where R and U are the coupling constants of the constructed effective theory, and L is the linear size of the system. Some simple universal results are obtained.

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I. INTRODUCTION

The theory of continuous phase transitions is based on the hypotheses that at temperatures close to the critical T_c, there is only one dominating length scale related to the critical behavior of the system. Because of the divergent nature of the correlation length as the critical point is approached, the microscopic details of the system becomes irrelevant for the critical exponents describing the singular dependence of the thermodynamic functions. This intuitive picture is based on the grounds of the renormalization-group treatment of second-order phase transitions.

Scaling is a central idea in critical phenomena near a continuous phase transition and in the field theory when we are interested in the continuum limit [1]. In both cases we are interested in the singular behavior emerging from the overwhelming large number of degrees of freedom, corresponding to the original cutoff scale, which need to be integrated out leaving behind long wavelength which vary smoothly. Their behavior is controlled by a dynamically generated length scale: the correlation length ξ_b. Such a fundamental idea is difficult to test theoretically because it requires a study of a huge number of interacting degrees of freedom. Experimentally, however, one hopes to be able to study scaling in finite systems near a second-order phase transition. Namely, the system is confined to a finite geometry and the finite-size scaling theory is expected to describe the behavior of the system near the bulk critical temperature (for a review on the finite-size scaling theory see Refs. [2] and [3]).

The $\mathcal{O}(n)$-symmetric vector models are extensively used to explore the finite-size scaling theory, using different methods and techniques both analytically and numerically. The most thoroughly investigated case is the particular one corresponding to the limit $n=\infty$ (this limit includes also the mean spherical model) [3]. In this limit, these models are exactly soluble for arbitrary dimensions and in a general geometry. These investigations were devoted exclusively to systems with short (including nearest neighbors) as well as long-range forces decaying with the interparticle distance in a power law. For finite n the most frequently used analytical method is that of renormalization group [1,4]. However this is limited to the case of short-range interaction. The crossover from long to short-range forces was discussed in Ref. [5], where it has been found that renormalized values of the temperature and the coupling constant are continuous functions of the parameter controlling the range of the interaction, when this approaches the value 2 characterizing the short-range force potential. The case of pure long-range interaction was investigated very recently in Ref. [6] (a comment on the method and the results obtained there is presented in Sec. IV). In the mean time, special attention was devoted to the investigation of finite-size scaling for the mean-spherical model with long-range interaction (for a review see Ref. [7] and references therein).

In recent years there has been an increasing interest in the numerical investigation of the critical properties of systems with long-range interaction decaying at large distances r by a power law as $r^{-d-\sigma}$, where d is the space dimensionality and σ is the parameter controlling the range of the interaction. The mostly used technique for this achievement is the Monte Carlo method. This method was used to investigate the critical properties of Heisenberg ferromagnetic systems [8] as well as Ising models [9,10]. Nevertheless all the analysis there was concentrated on systems with classical critical behavior in the sense that the critical exponents are given by Landau theory.

In this paper we present a detailed investigation of the finite-size scaling properties of the field theoretic $\mathcal{O}(n)$ vector φ^4 model with long-range interaction. We will also check the influence of the interaction range on the critical behavior. These interactions enter the exact expressions for the free energy only through their Fourier transform, where the leading asymptotic is $U(q)\sim q^{\sigma^*}$, and $\sigma^*=\min(\sigma,2)$ [11]. As it was shown for bulk systems by renormalization-group arguments, $\sigma\geqslant 2$ corresponds to the case of finite (short-) range interactions, i.e., the universality class then does not depend on σ [11,12]. Values satisfying $0<\sigma<2$ correspond to long-range interactions and the critical behavior depends on σ. With the renormalization-group treatment it has been found that the critical behavior depends on the small parameter $\varepsilon=2\sigma-d$, where 2σ corresponds to the upper critical dimension[11]. According to the above reasoning one usually con-

*Email address: chamati@issp.bas.bg
†Email address: tonchev@issp.bas.bg

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siders the case $\sigma>2$ as uninteresting for critical effects, even for the finite-size treatments [13]. So, here we will consider only the case $0<\sigma\leqslant 2$.

Here, we will provide a systematic and controlled approach to the quantitative computation of the thermodynamic momenta, usually used in numerical analysis. These momenta are related to the Binder's cumulant and to various thermodynamic functions like the susceptibility. We will concentrate on the scaling properties of the coupling constants defining the system in the vicinity of the critical point. Our method is quite general and should apply to a large extent on the investigation of finite-size scaling in systems with long-range interaction in the vicinity of the critical point.

The plan of the paper is as follows. In Sec. II we review, briefly, the φ^4 model with long-range interaction and discuss its bulk critical behavior. Section III is devoted to the explanation of the methods used here to achieve our analysis. We end the section with the computation of some thermodynamic quantities of interest. In Sec. IV we discuss our results briefly. In the remainder of the paper we present some details of the calculations of some formula used throughout the paper.

II. FINITE-SIZE SCALING FOR SYSTEMS WITH LONG-RANGE INTERACTIONS

In the vicinity of its critical point the Heisenberg model, with long-range interaction decaying as power law, is equivalent to the d-dimensional $\mathcal{O}(n)$-symmetric model [14]

$$\beta\mathcal{H}\{\varphi\}=\frac{1}{2}\int_V d^dx\left[(\nabla^{\sigma/2}\varphi)^2+r_0\varphi^2+\frac{1}{2}u_0\varphi^4\right], \quad (2.1)$$

where φ is a short hand notation for the space dependent n-component field $\varphi(x)$, $r_0=r_{0c}+t_0$ ($t_0\propto T-T_c$) and u_0 are model constants. V is the volume of the system. In Eq. (2.1), we assumed $\hbar=k_B=1$ and the size scale is measured in units in which the velocity of excitations $c=1$. We note that the first term in the model denotes $\boldsymbol{k}^\sigma|\varphi(\boldsymbol{k})|^2$ in the momentum representation, where the parameter $0<\sigma\leqslant 2$ takes into account short-range as well as long-range interactions. β is the inverse temperature. The nature of the spectrum suggests that the critical exponent $\eta=2-\sigma$ [11,12]. Here we will consider periodic boundary conditions. This means

$$\varphi(x)=\frac{1}{\sqrt{V}}\sum_{\boldsymbol{k}}\varphi(\boldsymbol{k})\exp(i\boldsymbol{k}\cdot x), \quad (2.2)$$

where $\boldsymbol{k}$ is a discrete vector with components $k_i=2\pi n_i/L$ ($n_i=0,\pm 1,\pm 2,\ldots,\ i=1,\ldots,d$) and a cutoff $\Lambda\sim a^{-1}$ (a is the lattice spacing). In this paper, we are interested in the continuum limit, i.e., $a\to 0$. As long as the system is finite we have to take into account the following assumptions $L/a\to\infty$, $\xi_b\to\infty$ while ξ_b/L is finite.

Fisher *et al.* [11] and Yamazaki and Suzuki [12] have shown that for the model under consideration the Landau theory holds for $d>2\sigma$. In the opposite case, i.e., $d<2\sigma$ an expansion in powers of $\varepsilon=2\sigma-d=4-d-2\eta$ takes place, where 2σ plays the role of the upper critical dimension. We will present the renormalized parameters that characterize the bulk critical behavior and appear in the scaling functions. Since the computations are standard [1], we will be quite brief.

The application of the renormalized theory, above the critical temperature, to the model Hamiltonian requires a scaling field amplitude Z, a coupling constant renormalization Z_g, and a renormalization of the φ^2 insertions in the critical theory Z_t. In terms of these, we define as usual

$$t=ZZ_t^{-1}(r_0-r_{0c}) \quad \text{and} \quad g=\mathrm{l}^{-\varepsilon}Z^2Z_g^{-1}u_0. \quad (2.3)$$

In the remainder we will work in units where the reference length l is set to unity. To one loop order the renormalization constants in the minimal subtraction scheme are given by [12]

$$Z=1+\mathcal{O}(\hat{g}^2), \quad (2.4a)$$

$$Z_t=1+\frac{n+2}{\varepsilon}\hat{g}+\mathcal{O}(\hat{g}^2), \quad (2.4b)$$

$$Z_g=1+\frac{n+8}{\varepsilon}\hat{g}+\mathcal{O}(\hat{g}^2). \quad (2.4c)$$

In Eqs. (2.4),

$$\hat{g}=g\frac{2}{(4\pi)^{d/2}\Gamma(d/2)}=\frac{2g}{(4\pi)^{\sigma}\Gamma(\sigma)}\times\left(1+\frac{\varepsilon}{2}[\ln(4\pi)+\psi(\sigma)]+\mathcal{O}(\varepsilon^2)\right), \quad (2.5)$$

where $\psi(x)$ is the digamma function.

The fixed point of the β function is at $\hat{g}=\hat{g}^*$ with

$$\hat{g}^*=\frac{\varepsilon}{n+8}+\mathcal{O}(\varepsilon^2). \quad (2.6)$$

Before starting to investigate the finite-size scaling in the field theoretical model under consideration, we shall recall briefly the corresponding renormalization-group formalism. In the continuum limit, the lattice spacing completely disappears. The integration over wave vectors of the fluctuations are evaluated without cutoff and are convergent. When some dimensions of the system are finite the integrals over the corresponding momenta are transformed into sums. Since the lattice spacing is taken to be zero, the limits of the sums still tend to infinity.

From general renormalization-group considerations an observable X, the susceptibility for example, will scale like [15]:

$$X[t,g,\mathrm{l},L]=\zeta(\rho)X[t(\rho),g(\rho),\mathrm{l}\rho,L], \quad (2.7)$$

where t is the reduced temperature, g a dimensionless coupling constant, and L the finite-size scale. The length scale l is introduced in order to control the renormalization procedure.

It is known that in the bulk limit, when $g(\rho)$ approaches its stable fixed point g^*, we have

$$t(\rho)\approx t\rho^{1/\nu} \quad \text{and} \quad \zeta(\rho)\approx\rho^{\gamma_x/\nu}, \tag{2.8}$$

where γ_x and ν are the bulk critical exponents measuring the divergence of the observable X and the correlation length, respectively, in the vicinity of the critical point and ρ is a scaling parameter. Using dimensional analysis together with Eq. (2.7) one gets

$$X[t,g,\mathrm{l},L]=\zeta(\rho)X[t(\rho)(\rho\mathrm{l})^2,g(\rho),1,L/\mathrm{l}\rho]. \tag{2.9}$$

Choosing the arbitrary parameter $\rho=L/\mathrm{l}$, we obtain the well-known finite-size scaling result

$$X[t,g,\mathrm{l},L]=L^{\gamma_x/\nu}f(tL^{1/\nu}). \tag{2.10}$$

Here the function $f(x)$ is a universal function of its argument. In the remainder of this paper we will verify the scaling relation (2.10) in the framework of model (2.1).

III. FINITE-SIZE SCALING BELOW THE UPPER CRITICAL DIMENSION

A. Method

The method, we shall use here to analyze the finite-size scaling of the model under consideration, is originally due to Lüscher [16] in his study on the quantum $\mathcal{O}(n)$ nonlinear σ model in $1+1$ dimensions. An extension of the method was employed by Brezin and Zinn-Justin [17] and by Rudnick *et al.* [18] in their works on the finite-size scaling in systems with short-range potentials. Very recently it was used in the investigation of crossovers in quantum $\mathcal{O}(n)$ systems near their upper critical dimension [19]. We will see here that the problem related to finite-size scaling in systems with long-range forces can be successfully analyzed by the same approach. Nevertheless, here we will observe the emergence of some subtleties, which need to be discussed.

The central idea of the method is that at finite linear size L of the system, one can treat the $\boldsymbol{k}=0$ mode of the field $\varphi(x)$, playing the role of the magnetization, separately from the nonzero $\boldsymbol{k}$ modes. The nonzero modes are treated perturbatively using the loop expansion. They are integrated out to yield an effective Hamiltonian for the lowest mode only. All the modes being integrated out are regulated in the infrared by $|\boldsymbol{k}|^\sigma$ and consequently the process is necessarily free of infrared divergences. On the other hand the renormalizations of the bulk theory control the ultraviolet divergences at finite size. In other words if we define by

$$\phi=\frac{1}{V}\int_V d^dx\,\varphi(x) \tag{3.1}$$

the total spin by unit volume, then, from $\mathcal{H}$, we can get an effective Hamiltonian function of ϕ after entirely integrating out the $\varphi(k\neq 0)$ fields:

$$\mathcal{H}_{\mathrm{eff}}=\frac{L^d}{2}\left(R\phi^2+\frac{U}{2}\phi^4\right). \tag{3.2}$$

The coupling constants R and U are computed in powers of ε, with the initial coupling constants renormalized as in their bulk critical theory. This approach will rule out all the ultraviolet divergences of the bulk critical point. The new coupling constants are necessarily free of all ultraviolet divergences since the theory is super-renormalizable [12]. They are also free of infrared divergences as we are only integrating out finite modes. Obviously, these constants must obey the scaling forms,

$$R=L^{\eta-2}f_R(tL^{1/\nu}) \quad \text{and} \quad U=L^{d-4+2\eta}f_U(tL^{1/\nu}) \tag{3.3}$$

for $t\gtrsim 0$, where f_R and f_U are scaling functions that are properties of the bulk critical point. They are analytic at $t=0$. This is a consequence of the fact that only finite modes have been integrated out.

Once the scaling functions f_R and f_U are known, one can attack the problem of computing observables in the φ^4 theory with the action $\mathcal{H}_{\mathrm{eff}}$. This theory is in dimension d close to the upper critical dimension 2σ (not in d close to the usual 4), and the problem seems to be unsolvable. In the next section we will show that it is not the case.

In order to investigate the long-distance physics of the finite system, one has to calculate thermal averages with respect to the new effective Hamiltonian defined in Eq. (3.2). They are related to the thermodynamic functions of the system under consideration. The averages of the field ϕ are defined by

$$\mathcal{M}_{2p}=\langle(\phi^2)^p\rangle=\frac{\int d^n\phi\,\phi^{2p}\exp(-\mathcal{H}_{\mathrm{eff}})}{\int d^n\phi\exp(-\mathcal{H}_{\mathrm{eff}})}. \tag{3.4}$$

Using an appropriate rescaling of the field ϕ: $\Phi=(UL^d)^{1/4}\phi$, we can transform the effective Hamiltonian into

$$\mathcal{H}_{\mathrm{eff}}=\frac{1}{2}z\Phi^2+\frac{1}{4}\Phi^4, \tag{3.5}$$

where the scaling variable $z=RL^{d/2}U^{-1/2}$ is an important quantity in the investigations of finite-size scaling in critical statics [17,18] as well as in critical dynamics [20,21]. With the effective Hamiltonian (3.5), we obtain the general scaling relation

$$\mathcal{M}_{2p}=L^{-p(d-2+\eta)}\frac{L^{p(d-4+2\eta)/2}}{U^{p/2}}f_{2p}\left(RL^{2-\eta}\frac{L^{(d-4+2\eta)/2}}{U^{1/2}}\right) \tag{3.6}$$

for the momenta of the field ϕ. Having in mind Eqs. (3.3), we can write down Eq. (3.6) in the following scaling form

$$\mathcal{M}_{2p}=L^{-p(d-2+\eta)}\mathcal{F}_{2p}(tL^{1/\nu}), \tag{3.7}$$

in agreement with the finite-size scaling predictions of Eq. (2.10). In Eq. (3.7), the functions $\mathcal{F}_{2p}(x)$ are universal.

All the measurable thermodynamic quantities can be obtained from the momenta $\mathcal{M}_{2p}$. For example the susceptibility is obtained from

$$\chi=\frac{1}{n}\int_V d^dx\langle\varphi(x)\varphi(0)\rangle=L^{2-\eta}\mathcal{F}_2(tL^{1/\nu}). \tag{3.8}$$

Another quantity of importance for numerical analysis of the finite-size scaling theory is the Binder's cumulant defined by

$$B=1-\frac{1}{3}\frac{\mathcal{M}_4}{\mathcal{M}_2^2}. \tag{3.9}$$

In the remainder of this section we concentrate on the computation of the coupling constants R and U of the effective Hamiltonian (3.2) for the system with long-range interaction decaying with the distance as a power law. As a consequence we will deduce results for the characteristic variable $z=RU^{-1/2}L^{2-\eta-\varepsilon/2}$, the susceptibility χ, and the amplitude ratio $r=\mathcal{M}_4/\mathcal{M}_2^2$ entering the definition of the Binder's cumulant.

B. Computation of the coupling constants R and U

As we explained above, loop corrections will be treated perturbatively on the nonzero $\boldsymbol{k}$ modes. At the tree level (lowest order in $\varepsilon=2\sigma-d$) this procedure generates a shift of the critical temperature T_c and a change of the coupling constant u_0 and additional operators involving powers of φ larger than 4. The calculations will be performed in the renormalized theory. The renormalized coupling constant u_R is expressed in terms of the dimensionless coupling constant $g=\mathrm{l}^{\varepsilon}u_R$ in which the parameter l is an arbitrary length scale. Here we will work in system in which $\mathrm{l}=1$. Throughout these calculations we use the minimal subtraction scheme. In this scheme, the counterterms of the massless theory including the φ^2 insertions are introduced. The one-loop counterterm for the coupling constant and the φ^2 insertion will be the only one relevant in the lowest corrections.

The finite-size correction to the renormalized coupling constant t is given by

$$\mathcal{W}^t_{d,\sigma}(t,g,L)=(n+2)g\frac{1}{L^d}{\sum_{\boldsymbol{k}}}'\frac{1}{t+|\boldsymbol{k}|^\sigma} \tag{3.10}$$

to one-loop order.

In order to investigate the finite-size scaling of the model under consideration one can use a suitable approach allowing to simplify the analytical calculations. In the case $\sigma=2$ it is possible to replace the summand by its Laplace transform. This is the so-called Schwinger representation. The aim of this approach is to reduce the d-dimensional sum in the right-hand side of Eq. (3.10) to the one-dimensional effective problem. In the general case of arbitrary σ, one cannot just use the Schwinger transformation or at least in its familiar form. So we have to solve the problem by introducing some kind of generalization for it. In the spirit of the same problem a method to investigate the finite-size scaling in the framework of the mean-spherical model was suggested in Ref. [22]. The method is based upon the following genius identity

$$\frac{1}{1+z^\alpha}=\int_0^\infty dx\,\exp(-xz)x^{\alpha-1}E_{\alpha,\alpha}(-x^\alpha), \tag{3.11a}$$

where the functions

$$E_{\alpha,\beta}(z)=\sum_{\mathrm{l}=0}^{\infty}\frac{z^{\mathrm{l}}}{\Gamma(\alpha\mathrm{l}+\beta)} \tag{3.11b}$$

is the so-called Mittag-Leffler type functions. For a more recent review on these functions and others related to them, and their application in statistical and continuum mechanics see Ref. [23]. See also Ref. [22] and Appendix A.

Using the identity (3.11), one gets, after some algebra,

$$\mathcal{W}^t_{d,\sigma}(t,g,L)=(n+2)g\frac{L^{\sigma-d}}{(2\pi)^\sigma}\int_0^\infty dx x^{\sigma/2-1}E_{\sigma/2,\sigma/2}\left(-x^{\sigma/2}\frac{tL^\sigma}{(2\pi)^\sigma}\right)[\mathcal{A}^d(x)-1], \tag{3.12a}$$

where

$$\mathcal{A}(x)=\sum_{\mathrm{l}=-\infty}^{\infty}e^{-x\mathrm{l}^2}. \tag{3.12b}$$

The analytic properties of the function $\mathcal{A}(x)$ are known very well. For large x, $\mathcal{A}(x)-1$ decreases exponentially and the integral in the right-hand side of Eq. (3.12a) converges at infinity. For small x, the Poisson transformation $\mathcal{A}(x)=(\pi/x)^{1/2}\mathcal{A}(\pi^2/x)$ shows that $\mathcal{A}(x)$ converges.

For small x the integral in the right-hand side of Eq. (3.12a) has ultraviolet divergence for $\mathrm{Re}\,d>\sigma$. So, an analytic continuation in d is required to give a meaning to the integral. Adding and subtracting the small asymptotic behavior of the function $\mathcal{A}(x)$, we get after some algebra,

$$\mathcal{W}^t_{d,\sigma}(t,g,L)=(n+2)g\frac{L^{\sigma-d}}{(2\pi)^\sigma}F_{d,\sigma}(tL^\sigma)+2\pi(n+2)gL^{\sigma-d}\left[(4\pi)^{d/2}\Gamma\left(\frac{d}{2}\right)\sigma\sin\frac{d\pi}{\sigma}\right]^{-1}(tL^\sigma)^{d/\sigma-1}, \tag{3.13a}$$

where

$$F_{d,\sigma}(y)=\int_0^\infty dx x^{\sigma/2-1}E_{\sigma/2,\sigma/2}\left(-\frac{yx^{\sigma/2}}{(2\pi)^\sigma}\right)\times\left[\mathcal{A}^d(x)-1-\left(\frac{\pi}{x}\right)^{d/2}\right]. \quad (3.13b)$$

In the particular case $\sigma=2$, from Eq. (3.13) we recover the result of Ref. [17].

By introducing the φ^2 counterterm insertion the renormalized coupling constant t is replaced by tZ_t, where Z_t is given by Eq. (2.4b). Hence to one loop order we have

$$R=t\left(1+\hat{g}\frac{n+2}{\varepsilon}\right)+\mathcal{W}^t_{d,\sigma}(t,g,L). \quad (3.14)$$

At $d=2\sigma$, $\mathcal{W}^t_{d,\sigma}(t,g,L)$ has a simple pole. An expansion about this pole leads to the final expression

$$R=t+\frac{n+2}{\sigma}\hat{g}t\ln t+2^{\sigma-1}(n+2)\Gamma(\sigma)\hat{g}L^{-\sigma}F_{2\sigma,\sigma}(tL^\sigma)+\mathcal{O}(\hat{g}^2). \quad (3.15)$$

This result shows that, at the critical point, R has the required scaling properties of Eq. (3.3), since

$$\nu^{-1}=\sigma-\frac{n+2}{n+8}\varepsilon+\mathcal{O}(\varepsilon^2). \quad (3.16)$$

For the finite system the renormalized coupling constant g, to one-loop order, is shifted by a quantity expressed in the form

$$\mathcal{W}^g_{d,\sigma}(t,g,L)=-(n+8)g^2\frac{1}{L^d}{\sum_{\mathbf{k}}}'\frac{1}{(t+|\mathbf{k}|^\sigma)^2}. \quad (3.17)$$

As one can see the summand here can be expressed as the first derivative of the summand of Eq. (3.10) with respect to t. So, the result for U can be derived from that of R. Using this fact one gets

$$\mathcal{W}^g_{d,\sigma}(t,g,L)=(n+8)g^2\left[\frac{L^{2\sigma-d}}{(2\pi)^\sigma}F'_{d,\sigma}(tL^\sigma)-L^{2\sigma-d}\frac{2}{\sigma(4\pi)^{d/2}}\times\frac{\Gamma(2-d/\sigma)\Gamma(d/\sigma)}{\Gamma(d/2)}(tL^\sigma)^{d/\sigma-2}\right], \quad (3.18)$$

where the prime indicates that we have the derivative of the function F with respect to its argument.

At the fixed point one ends up with

$$U=g\left[1+\hat{g}\frac{n+8}{\sigma}(1+\ln t)+\hat{g}\frac{n+8}{2^{1-\sigma}}\Gamma(\sigma)F'_{2\sigma,\sigma}(tL^\sigma)+\mathcal{O}(\hat{g}^2)\right] \quad (3.19)$$

for the renormalized coupling constant U. Equation (3.19) is obtained using the fact that at one-loop order the coupling constant is renormalized by Z_g form Eq. (2.4c). The obtained expression (3.19) shows that the coupling constant U obeys the scaling law of Eq. (3.3). Note that U has a finite limit as $t\to 0$, i.e. it is analytic at the bulk critical temperature. Indeed as $t\to 0$ one can use the expansion of the function $F_{2\sigma,\sigma}(y)$ for small y given by (see Appendix B)

$$F_{2\sigma,\sigma}(y)=F_{2\sigma,\sigma}(0)+2^{-\sigma}y\mathcal{C}_\sigma-\frac{2^{1-\sigma}}{\sigma\Gamma(\sigma)}y\ln y+\mathcal{O}(y^2), \quad (3.20a)$$

where

$$\mathcal{C}_\sigma=\frac{1}{\Gamma(\sigma)}\int_0^\infty\frac{du}{u}\left[E_{\sigma/2,1}\left(-\frac{u^{\sigma/2}}{(2\pi)^\sigma}\right)-\frac{u^\sigma}{\pi^\sigma}\mathcal{A}^{2\sigma}(u)+\frac{u^\sigma}{\pi^\sigma}\right]. \quad (3.20b)$$

After substitution of Eq. (3.20) in Eq. (3.19) the terms proportional to log$y$ cancel, which shows that the coupling constant U is finite at $t=0$. One gets

$$U=gL^{-\varepsilon}\left[1+\hat{g}\frac{n+8}{\sigma}\left(1+\frac{\sigma}{2}\Gamma(\sigma)\mathcal{C}_\sigma\right)+\mathcal{O}(\hat{g}^2)\right]$$

showing that U is analytic, as it should be, at the critical point.

C. Some thermodynamic quantities

1. Shift of the critical point

It is obvious that the coupling constant R in the effective Hamiltonian (3.2) is just the deviation of the temperature of the system from its ''critical'' value. By setting $t=0$ in (3.15), we obtain an expression for the finite-size shift of the bulk critical temperature T_c. This is given by

$$T_c-T_c(L)=\varepsilon 2^{\sigma-1}\frac{n+2}{n+8}\Gamma(\sigma)L^{-\sigma}F_{2\sigma,\sigma}(0), \quad (3.21)$$

where the coefficient $F_{2\sigma,\sigma}(0)$, appearing in the right-hand side of Eq. (3.21) can be evaluated for some particular values of the interparticle interaction range σ (Ref. [24])

$$F_{2\sigma,\sigma}(0)=\begin{cases}2\zeta(1/2), & \sigma=1/2,\\ 4\zeta(1/2)\beta(1/2), & \sigma=1,\\ -4.822\,719\,93, & \sigma=3/2,\\ -8\ln 2, & \sigma=2.\end{cases} \quad (3.22)$$

Here $\zeta(x)$ is the Riemann zeta function with $\zeta(\frac{1}{2}) = -1.460\,354\,508\ldots$ and $\beta(x)$ is the analytic continuation of the Dirichlet series:

$$\beta(x)=\sum_{l=0}^{\infty}\frac{(-1)^l}{(2l+1)^x},$$

with $\beta(\frac{1}{2})=0.667\,691\,457\ldots$. Remark that the function $F_{2\sigma,\sigma}(0)$ increases as the parameter σ vanishes.

In fact, since there is no true phase transition in the finite system, the critical temperature is shifted to a ''pseudocritical'' temperature, $T_c(L)$, corresponding to the rounding of the thermodynamic singularities holding in the bulk limit. From Eq. (3.21) one remarks that $T_c(L)$ is larger than T_c, confirming previously obtained results in the framework of the spherical model [25]. Notice also that for the shift exponent λ, we get $\lambda=\sigma$ to lowest order in ε.

2. Binder's cumulant

In this subsection we are interested in the calculation of the amplitude ratio $r=\mathcal{M}_4/\mathcal{M}_2^2$ instead of the Binder's cumulant from definition (3.9). This quantity can be expressed in power series of the scaling variable $z=RL^{2-\eta-\varepsilon/2}U^{-1/2}$ as

$$r=\frac{n}{4}\frac{\Gamma^2(\frac{1}{4}n)}{\Gamma^2[\frac{1}{4}(n+2)]}\left\{1-z\left[\frac{\Gamma[\frac{1}{4}(n+6)]}{\Gamma[\frac{1}{4}(n+4)]}+\frac{\Gamma[\frac{1}{4}(n+2)]}{\Gamma[\frac{1}{4}n]}-2\frac{\Gamma[\frac{1}{4}(n+4)]}{\Gamma[\frac{1}{4}(n+2)]}\right]+z^2\left[\frac{\Gamma[\frac{1}{4}(n+6)]\Gamma[\frac{1}{4}(n+2)]}{\Gamma[[\frac{1}{4}(n+4)]\Gamma[\frac{1}{4}n]}+3\frac{\Gamma^2[\frac{1}{4}(n+4)]}{\Gamma^2[\frac{1}{4}(n+2)]}-n-1\right]+\mathcal{O}(z^3)\right\}. \tag{3.23}$$

So, in order to obtain a result for r it is enough to evaluate z at the fixed point g^* and to deduce the value for the Binder's cumulant. As we mentioned before this parameter appears in all thermodynamic functions through the momenta defined earlier in this paper.

At the fixed point g^* in the vicinity of the upper critical dimension, we obtain

$$z^*\equiv\left.\frac{RL^{2-\eta}}{\sqrt{UL^{\varepsilon}}}\right|_{\text{fixed point}}=\frac{1}{\sqrt{g^*}}\left[y-\frac{\varepsilon}{2\sigma}y\left(1-\frac{n-4}{n+8}\ln y\right)+2^{\sigma-1}\varepsilon\frac{n+2}{n+8}\Gamma(\sigma)F_{2\sigma,\sigma}(y)-\varepsilon 2^{\sigma-2}y\Gamma(\sigma)F'_{2\sigma,\sigma}(y)\right]. \tag{3.24}$$

This result is obtained by using Eq. (3.16) and the fact that up to one loop order the terms proportional to $\ln L$ cancel. In Eq. (3.24), we introduce the scaling variable $y=tL^{1/\nu}$. Finally let us notice that from this equation one can see easily that z^* verifies the finite-size scaling hypotheses and consequently all the thermodynamic functions do.

At the critical temperature T_c (i.e., $t=0$, and so $y=0$), we obtain

$$z_0^*=\sqrt{\varepsilon}\left[\frac{n+2}{\sqrt{n+8}}\sqrt{\frac{\Gamma(\sigma)}{2\pi^{\sigma}}}F_{2\sigma,\sigma}(0)+\mathcal{O}(\varepsilon)\right]. \tag{3.25}$$

Numerical values for the amplitude ratio (3.23) can be obtained by replacing the value of z_0^* from Eq. (3.25) and taking some specific values of the small parameter ε. Note that the scaling variable z is proportional to $\sqrt{\varepsilon}$ as it was found previously (see Ref [17]. for example) in the case of short-range forces. Furthermore it coincides with the result of Ref [6]. for the scaling variable x in the case of long-range interaction. Consequently all the thermodynamic function will be computed in powers of $\sqrt{\varepsilon}$.

3. Magnetic susceptibility

As we mentioned earlier, there is no phase transition in the finite system under consideration. Consequently there will be no ''true'' correlation length. An expression for it can be deduced from that of the susceptibility (3.8) trough the relation:

$$\xi^{2-\eta}=\chi. \tag{3.26}$$

The analyticity of the susceptibility is a consequence of that the coupling constants R and U.

From Eq. (3.8) in the region $tL^{\sigma}\ll 1$ (i.e., $z\ll 1$), we obtain for the susceptibility

$$\chi=\frac{L^{\sigma}}{\sqrt{\varepsilon}}\frac{2\sqrt{2}}{\sqrt{(4\pi)^{\sigma}\Gamma(\sigma)}}\frac{\sqrt{n+8}}{n}\frac{\Gamma[\frac{1}{4}(n+2)]}{\Gamma\left(\frac{n}{4}\right)}\times\left[1-z\left(\frac{n}{4}\frac{\Gamma(n/4)}{\Gamma[\frac{1}{4}(n+2)]}-\frac{\Gamma[\frac{1}{4}(n+2)]}{\Gamma(n/4)}\right)+z^2\left(\frac{1-n}{4}+\frac{\Gamma^2[\frac{1}{4}(n+2)]}{\Gamma^2(n/4)}\right)-\hat{g}\frac{n+8}{\sigma}\left(1+\frac{\sigma}{2}\Gamma(\sigma)\mathcal{C}_{\sigma}\right)+\mathcal{O}(\hat{g}z,z^3)\right] \tag{3.27}$$

at the bulk critical point T_c.

To the lowest order in ε, after taking the limit $n\to\infty$, we find that the correlation length scales like

$$\xi \sim \varepsilon^{-1/2\sigma} L,$$

confirming the results obtained in the spherical model [26] and showing that this behavior is not a characteristic of the spherical limit, i.e., $n \to \infty$.

In the region $tL^{\sigma} \gg 1$ (i.e., $z \gg 1$), from Eq. (3.8), we get

$$\chi = \frac{1}{t}\Bigg[1 - \frac{n+2}{\sigma}\hat{g}\ln t - 2^{\sigma-1}(n+2)\Gamma(\sigma)\hat{g}(tL^{\sigma})^{-1} \times F_{2\sigma,\sigma}(tL^{\sigma}) - \frac{1}{2}(n+2)(4\pi)^{\sigma} \times \Gamma(\sigma)\hat{g}(tL^{\sigma})^{-2} + \mathcal{O}(\hat{g}^2)\Bigg]. \tag{3.28}$$

The function $F_{d,\sigma}(y)$ has the following large y asymptotic behavior (see Appendix B)

$$F_{d,\sigma}(y) \simeq -\frac{(2\pi)^{\sigma}}{y} + \frac{4^{\sigma}\pi^{\sigma-d/2}\Gamma[(d+\sigma)/2]}{y^2\Gamma(-\sigma/2)} {\sum_{l}}' \frac{1}{|\boldsymbol{l}|^{d+\sigma}} \tag{3.29a}$$

for the case $0 < \sigma < 2$, and

$$F_{d,2}(y) \simeq -\frac{4\pi^2}{y} + d(2\pi)^{(5-d)/2} y^{(d-3)/4} e^{-\sqrt{y}} \tag{3.29b}$$

for the particular case $\sigma = 2$. These results show that the last term in Eq. (3.28) is just canceled by the first term in Eqs. (3.29).

In the case of long-range interaction $0 < \sigma < 2$, we obtain for the susceptibility

$$\chi = \chi_{\infty}\Bigg[1 - \sigma\hat{g}(n+2)2^{3\sigma-2}(tL^{\sigma})^{-3}\frac{\Gamma(3\sigma/2)\Gamma(\sigma)}{\Gamma(1-\sigma/2)} \times {\sum_{l}}' \boldsymbol{l}^{-3\sigma} + \mathcal{O}(\hat{g}^2)\Bigg] \tag{3.30}$$

in agreement with the finite-size scaling hypothesis (3.8). Equation (3.30) shows that the finite-size scaling behavior of the system is dominated by the bulk critical behavior, with small correction in powers of L. First the power-law falloff of the finite-size corrections to the bulk critical behavior, due to long-range nature of the interaction, was found in the framework of the spherical model [27,28]. Here, we extended this result to finite n using a perturbative approach.

It should be noted that the above result [Eq. (3.30)] cannot be continued smoothly to the case of short-range interaction $\sigma = 2$, since then $F_{4,2}(y)$ [see Eq. (3.29b)] falls off exponentially fast and, correspondingly, the finite-size corrections to χ are exponentially small

$$\chi = \chi_{\infty}[1 - 8\hat{g}\sqrt{2\pi}(n+2)(tL^2)^{-3/4}e^{-\sqrt{t}L} + \mathcal{O}(\hat{g}^2)]. \tag{3.31}$$

In Ref. [29], it has been argued that the approach used in Refs. [6] and [17–21] is inadequate in the region $tL^2 \gg 1$ above T_c, for the lattice φ^4 model, and one should employ *the ordinary perturbation* approach, where *all modes* are treated perturbatively. In this way the following result, valid for arbitrary dimensions $2 < d < 4$, is obtained [see Eq. (4.4) of Ref. [29]]

$$\chi = \chi_{\infty}[1 - 4(n+2)u^* d(2\pi L/\xi_1)^{(1-d)/2}e^{-L/\xi_1} + \mathcal{O}(u^{*2})], \tag{3.32}$$

where u^* is the fixed point value of the renormalized coupling constant and ξ_1 is the bulk exponential correlation length [30].

Now we are in position to shed some light on this debate. Note that all our calculations are performed up to the order $g^1 \sim \varepsilon^1$. One can easily see, by setting $d = 4 - \varepsilon$ in Eq. (3.32), that both results Eq. (3.31) and Eq. (3.32) coincide (if $\hat{g} = u^*/2\pi^2$) and any controversy gets down, at least up to the order ε^1. It is interesting to see what happens in higher order, e.g., do the powerlike terms cancel completely up to the second loop or higher orders. In this case, however, we need to have at our disposal the corresponding high-order terms in Eqs. (3.15) and (3.19). Indeed it is beyond the scope of the present paper.

IV. CONCLUSIONS

In this paper, we have investigated the finite-size scaling properties in the $\mathcal{O}(n)$-symmetric φ^4 model with long-range interaction potential decaying algebraically with the interparticle distance. We have found that the methods developed in Refs. [16–19] can be successfully extended to systems with long-range interaction by combining them with other known techniques. These techniques allow the investigation to be simplified and express the results for various thermodynamic functions in terms of simple and known mathematical functions.

Here we restricted our calculations to the critical domain $T \gtrsim T_c$ and investigated the model in dimensions less than the upper critical one, which turns out to be 2σ $(0 < \sigma \leqslant 2)$. We constructed an effective Hamiltonian, from the initial one, with new coupling constants R and U. These constants obey the scaling hypothesis (3.3). We found that the even momenta of the field φ, related to the thermodynamics of the finite system, are scaling functions of the characteristic variable

$$z = RU^{-1/2}L^{2-\eta-\varepsilon/2}.$$

This variable has the required scaling form predicted by the finite-size scaling theory. From the obtained forms of the constant R and U one concludes that z is a universal quantity, which does not depend of the details of the model.

We evaluated the finite-size shift, the susceptibility and the amplitude ratio $r = \mathcal{M}_4/\mathcal{M}_2^2$ at the tree level (lowest order in ε). We observed that the critical behavior of the system is dominated by its bulk critical behavior away from the critical domain and that the finite-size scaling is relevant in the vicinity of the critical point. The amplitude ratio r is evaluated as an expansion in powers of $z \sim \sqrt{\varepsilon}$. Our result is in consistency with that of Ref. [6]. But it is disagreement with the numerical results of the same paper. There, it has been found, using the Monte Carlo method, that r has an expansion in ε instead of its square root. At this time, we do not have a reasonable explanation of this fact. It is also pos-

sible that higher order in ε could improve the result. An amelioration of the result could also come from accounting finite cutoff effects, which were to be relevant in the investigation of finite systems and the comparison of the results with numerical works [31,32]. However this is the subject of another publication.

Notice that in the only work devoted to the exploration of finite-size scaling in $\mathcal{O}(n)$ systems with long-range interaction (Ref. [6]) the pertinent integrals have to be evaluated only numerically, due to the choice of a parametrization that does not reduce the *d*-dimensional problem to the effective one-dimensional one. The approach we used here is more efficient in the sense that the corresponding final expressions can be handled by analytical means. Consequently, we cannot make a direct comparison between the results of this paper and those obtained there.

Let us note that it would be interesting and useful to extend the result obtained here in the static limit to models including dynamics, since we believe that this is closely related to the extensively investigated field of quantum critical points, i.e., phase transitions occurring at zero temperature. In particular we find it useful to investigate the critical dynamic of the quantum model considered in Ref. [33] in the large *n* limit.

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APPENDIX A: SOME PROPERTIES OF THE MITTAG-LEFFLER TYPE FUNCTIONS

The Mittag-leffler type functions are defined by the power series [23]:

$$E_{\alpha,\beta}(z)=\sum_{k=0}^{\infty}\frac{z^k}{\Gamma(\alpha k+\beta)},\quad \alpha,\beta>0. \tag{A1}$$

They are entire functions of finite order of growth. Let us mention that the function corresponding the particular case $\beta=1$ was introduced by Mittag-Leffler. These function are very popular in the field of fractional calculus (for a recent review see Ref. [23]).

One of the most striking properties of these functions is that they obey the following useful identity [23]:

$$\frac{1}{1+z}=\int_0^{\infty}dx e^{-x}x^{\beta-1}E_{\alpha,\beta}(-x^{\alpha}z), \tag{A2}$$

which is obtained by means of term-by-term integration of the series (A1). The integral in Eq. (A2) converges in the complex plane to the left of the line $\mathrm{Re}z=1^{1/\alpha}$, $|\mathrm{arg}z|\leqslant\frac{1}{2}\alpha\pi$. The identity (A2) lies in the basis of the mathematical investigation of finite-size scaling in the spherical model with algebraically decaying long-range interaction (see Ref. [7] and references therein).

In some particular cases the functions $E_{\alpha,\beta}(z)$ reduce to known functions. For example, in the case corresponding to the short-range case we have

$$E_{1,1}(z)=\exp(z). \tag{A3}$$

Setting $z=y^{-\alpha}$, $y>0$ and $x=ty$, we obtain the Laplace transform

$$\frac{y^{\alpha-\beta}}{1+z^{\alpha}}=\int_0^{\infty}dte^{-zt}t^{\beta-1}E_{\alpha,\beta}(-t^{\alpha}) \tag{A4}$$

from which we derive the identity (3.11) by setting $\beta=\alpha$.

The asymptotic behavior of the Mittag-Leffler functions is given by the Lemma: [34]

Let $0<\alpha<2$, β be an arbitrary complex number, and γ be a real number obeying the condition

$$\frac{1}{2}\alpha\pi<\gamma<\min\{\pi,\alpha\pi\}.$$

Then for any integer $p\geqslant 1$ the following asymptotic expressions hold when $|z|\to\infty$:

(1) At $|\mathrm{arg}z|\leqslant\gamma$,

$$E_{\alpha,\beta}(z)=\frac{1}{\alpha}z^{(1-\beta)/\alpha}e^{z^{1/\alpha}}-\sum_{k=1}^{\infty}\frac{z^{-k}}{\Gamma(\beta-\alpha k)}+\mathcal{O}(|z|^{-p-1}). \tag{A5}$$

(2) At $\gamma\leqslant|\mathrm{arg}z|\leqslant\pi$,

$$E_{\alpha,\beta}(z)=-\sum_{k=1}^{\infty}\frac{z^{-k}}{\Gamma(\beta-\alpha k)}+\mathcal{O}(|z|^{-p-1}). \tag{A6}$$

APPENDIX B: ASYMPTOTIC BEHAVIOR OF THE FUNCTION $F_{d,\sigma}(y)$

To obtain the small y behavior (3.20a) of the function $F_{d,\sigma}(y)$ we use the identity [22]

$$\ln\phi=\alpha\int_0^{\infty}\frac{dx}{x}[E_{\alpha,1}(-x^{\alpha})-E_{\alpha,1}(-yx^{\alpha})] \tag{B1}$$

and the definition of the function $F_{d,\sigma}(y)$:

$$F_{d,\sigma}(y)=\int_0^{\infty}dx x^{\sigma/2-1}E_{\sigma/2,\sigma/2}\left(-\frac{yx^{\sigma/2}}{(2\pi)^{\sigma}}\right)\times\left[\mathcal{A}^d(x)-1-\left(\frac{\pi}{x}\right)^{d/2}\right]. \tag{B2}$$

After some algebra one obtains:

$$F_{2\sigma,\sigma}(y)=F_{2\sigma,\sigma}(0)+2^{-\sigma}y\mathcal{C}_{\sigma}-\frac{2^{1-\sigma}}{\sigma\Gamma(\sigma)}y\ln y+\mathcal{O}(y^2), \tag{B3a}$$

where

$$\mathcal{C}_{\sigma}=\frac{1}{\Gamma(\sigma)}\int_0^{\infty}\frac{du}{u}\left[E_{\sigma/2,1}\left(-\frac{u^{\sigma/2}}{(2\pi)^{\sigma}}\right)-\frac{u^{\sigma}}{\pi^{\sigma}}\mathcal{A}^{2\sigma}(u)+\frac{u^{\sigma}}{\pi^{\sigma}}\right]. \tag{B3b}$$

To obtain the large y asymptotic behavior (3.29) of the function $F_{d,\sigma}(y)$ we rewrite Eq. (B2) in the form

$$F_{d,\sigma}(y)=\pi^{d/2}\int_0^\infty dx x^{\sigma/2-d/2-1}E_{\sigma/2,\sigma/2}\times\left(-\frac{yx^{\sigma/2}}{(2\pi)^\sigma}\right)\sum_l{}' e^{-\pi^2 l^2/x}-\int_0^\infty dx x^{\sigma/2-1}E_{\frac{\sigma}{2},\frac{\sigma}{2}}\left(-\frac{yx^{\sigma/2}}{(2\pi)^\sigma}\right). \quad \text{(B4)}$$

Using the identity

$$\int_0^\infty dx x^{\sigma/2-1}E_{\sigma/2,\sigma/2}(-x^{\sigma/2})=1, \quad \sigma>0, \quad \text{(B5)}$$

from the second term of Eq. (B4) we obtain the first terms of Eqs. (3.29a) and (3.29b), respectively.

Next taking into account Eq. (A6) or Eq. (A3) for the function $E_{\alpha,\beta}(z)$, and after subsequent integration in the first term of Eq. (B4), we obtain finally the asymptotic behavior given by Eqs. (3.29a) and (3.29b).

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